Automatic Control

Chapter six

PID controllers

By

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PID controllers

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- The controller choice is an important step in the control process because this element is responsible of reducing the error (e_{ss}), rise time and overshoot
- There are three main types of relations proceed in the controller:
  1. Proportional relation (P)
  2. Derivative relation (D)
  3. Integration relation (I)
- These three types represent the relationships between the input and the output of the controller itself
- The presence of the controller is important because it is the method used to control the error and reduce it

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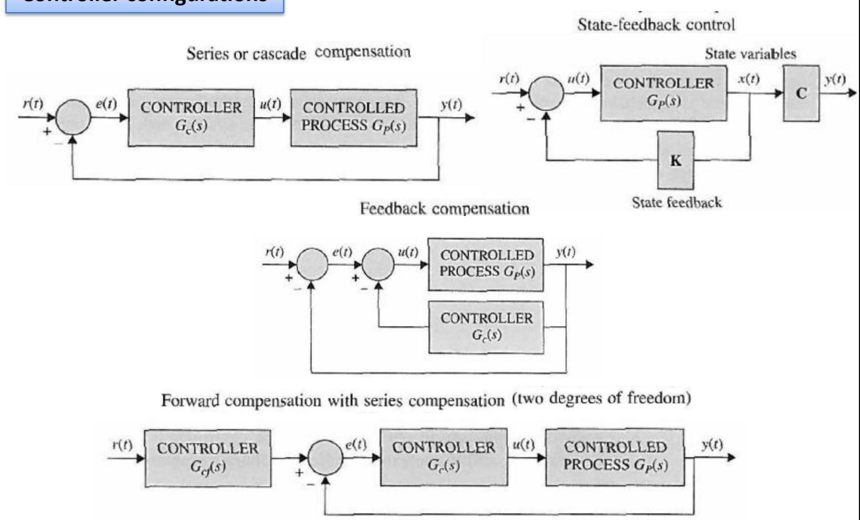
Controller configurations



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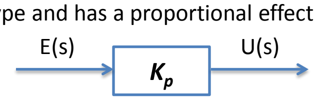
Proportional controller (P)

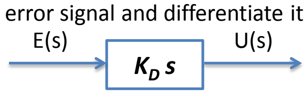
❑Is the simplest type and has a proportional effect on the output



❑A common type of this controller is the amplifier

Derivative controller (D)

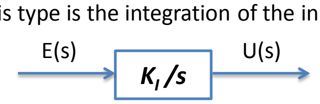
❑this type take the error signal and differentiate it



❑According to this relation, the output order is the order of the derivative of the input

Integrator controller (I)

❑The output in this type is the integration of the input



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Compression between types

	Proportional (P)	Integral (I)	Derivative (D)
Advantages	+ Fast response time + Minimizes fluctuation	+ Contains small offset + Returns system to steady state	+ Keeps system at consistent setting + Controls processes with rapidly changing outputs
Disadvantages	- Contains large offset - Does not bring system to desired set point	- Slow response time	- Slow response time - Requires combined use with another controller

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Examples

Controller	Estimates	When to use	Examples
P	Present	Systems with slow response, systems tolerant to offset	Float valves, thermostats, humidistat
I	Back	Not often used alone, as is too slow	Used for very noisy systems
D	Forward	Not used alone because it is too sensitive to noise and does not have set point	None
PI	Present & back	Often used	Thermostats, flow control, pressure control
PID	All time	Often used, most robust, but can be noise sensitive	Cases where the system has intertia that could get out of hand: i.e. temperature and concentration measurements on a reactor to avoid runaway.

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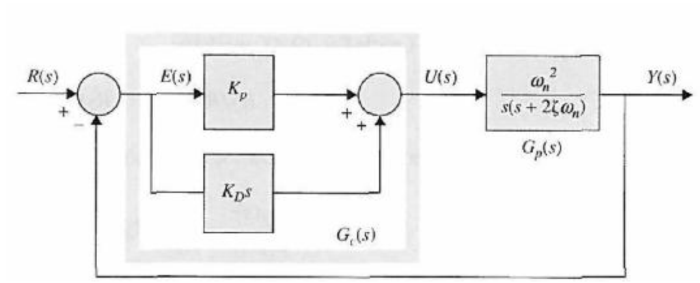
PD controller - Block diagram



$$G_c(s) = K_P + K_D s \qquad u(t) = K_P e(t) + K_D \frac{de(t)}{dt}$$

where K_P *and* K_D *are the proportional and derivative constants, respectively*

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Summary of Effects of PD Control

PD controller can affect the performance of a control system in the following ways:

1. Improving damping and reducing maximum overshoot.
2. Reducing rise time and settling time.
3. Increasing BW.
4. Improving GM, PM, and *Mr.*
5. *Possibly accentuating noise at higher frequencies.*
6. Possibly requiring a relatively large capacitor in circuit implementation.

PID controllers

A u t o m a t i c C o n t r o l

Design with PD Control

Example

Let us consider the following forward-path transfer function of a unity feedback control system

$$G(s) = \frac{4500K}{s(s+361.2)}$$

Let us set the performance specifications as follows:

- Steady-state error due to unit-ramp input < 0.000443
- Maximum overshoot < 5%
- Rise time $t_r < 0.005$ *sec*
- Settling time $t_s < 0.005$ *sec*

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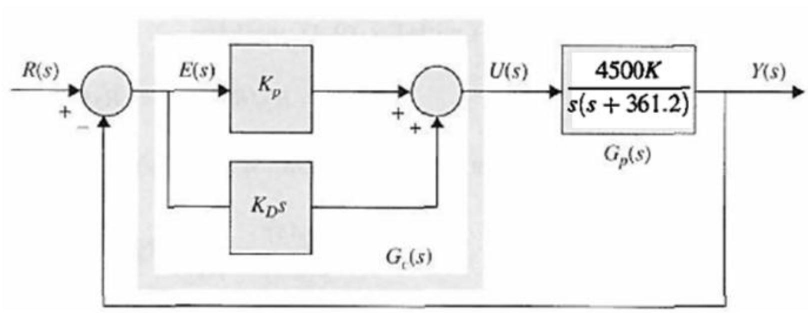
Design with PD Control

Solution

The system block diagram can be drawn as:



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Design with PD Control

Solution

We can start with satisfying the steady-state error which is due to unit-ramp input must be less than 0.000443

$$e_{ss} = \frac{R}{K_v}; K_v = \lim_{s \to 0} sG(s) \Rightarrow K_v = \lim_{s \to 0} \frac{4500K}{(s+361.2)} \approx 12.5K$$

$$e_{ss} = \frac{R}{K_v} = \frac{1}{12.5k} < 0.000443 \Rightarrow \mathrm{K} \geq 180.6$$

Assume k = 181 which satisfy the condition of steady-state error

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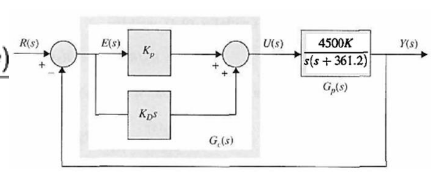
Design with PD Control

Solution

With the previous value of K and adding PD controller, he forward-path transfer function of the system becomes

$$G(s) = \frac{\Theta_y(s)}{\Theta_e(s)} = \frac{815,265(K_P + K_D s)}{s(s+361.2)}$$



The closed-loop transfer function is

$$\frac{\Theta_y(s)}{\Theta_r(s)} = \frac{815,265(K_P + K_D s)}{s^2 + (361.2 + 815,265 K_D)s + 815,265 K_P}$$

The characteristic equation is found as:

$$s^2 + (361.2 + 815,\ 265K_D)s + 815,\ 265K_P = 0$$

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Solution

To satisfy the other requirements :

- Maximum overshoot < 5%
- Rise time t_r < *0.005 sec*
- Settling time t_s < *0.005 sec*

we have to find the effect of the parameters K_D and K_P on the time response. The response can be found by applying Laplace inverse to the response in s-domain (i.e. $\Theta_y(s)$) which can be written as

$$\Theta_y(s) = \Theta_r(s)\,\frac{815,265(K_P + K_D s)}{s^2 + (361.2 + 815,265K_D)s + 815,265K_P}$$

$\Theta_r(s) = 1/s^2$.

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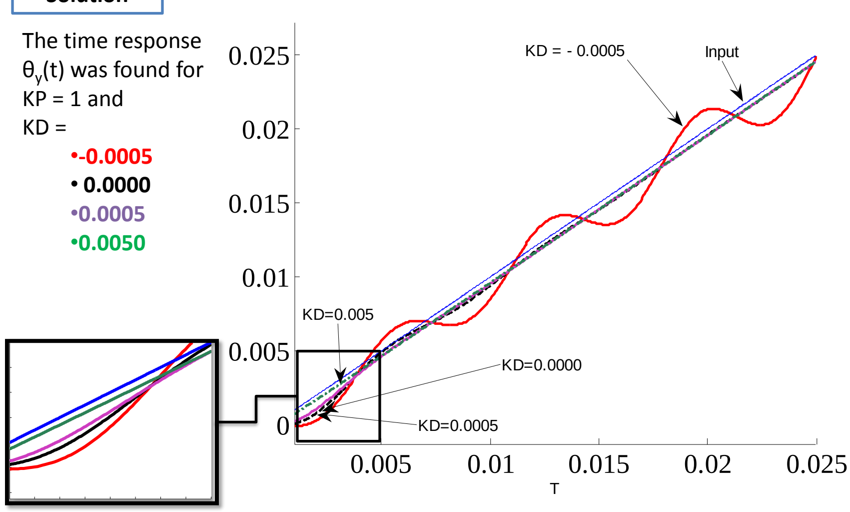
Solution

The time response $\theta_y(t)$ was found for KP = 1 and KD =

- **-0.0005**
- **0.0000**
- **0.0005**
- **0.0050**



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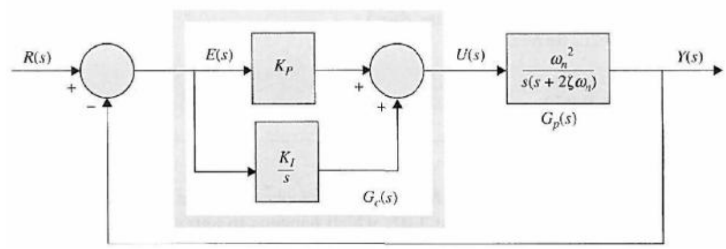
PI controller - Block diagram



$$G_c(s) = K_P + \frac{K_I}{s}$$

where K_P *and* K_I *are the proportional and integration constants, respectively*

PID controllers

Design with PI Control

Example

Let us consider the following forward-path transfer function of a unity feedback control system from the previous example

$$G(s) = \frac{4500K}{s(s+361.2)}$$

Let us set the performance specifications as follows:

- ❑Steady-state error due to parabolic input *t2us(t)/2 < 0.2*
- ❑Maximum overshoot < 5%
- ❑Rise time *t,- < 0.01 sec*
- ❑Settling time *ts < 0.02 sec*

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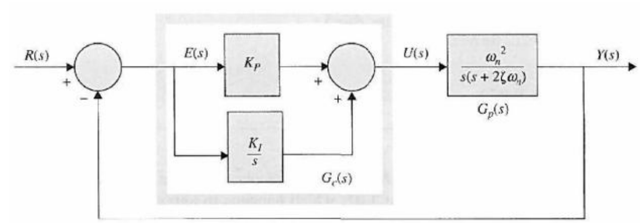
Solution



$$K_a = \lim_{s \to 0} s^2 G(s) = \lim_{s \to 0} s^2 \frac{4500KK_P(s + K_I/K_P)}{s^2(s + 361.2)} = \frac{4500KK_I}{361.2} = 12.46KK_I$$

$$e_{ss} = \frac{1}{K_a} = \frac{0.08026}{KK_I} (\leq 0.2)$$

Let us set *K = 181, simply because this was the value used in the previous example* Apparently, to satisfy a given steady-state error requirement for a parabolic input, the larger the *K, the smaller K_I can be. Solve for K_I for* the minimum steady-state error requirement of 0.2, we get the minimum value of K_I *to be* 0.002215. If necessary, the value of *K can be adjusted later.*

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Design with PI Control

Solution

With *K = 181, the characteristic equation of the closed-loop system is*

$$s^3 + 361.2s^2 + 815,265K_P s + 815,265K_I = 0$$

Applying Routh's test to *characteristic equation* yields the result that the system is stable for *0<K_I/Kp < 361.2*

$$\frac{K_I}{K_P} \ll 361.2$$

K_I/K_P	K_I	K_P	Maximum Overshoot (%)	t_r (sec)	t_s (sec)
0	0	1.00	52.7	0.00135	0.015
20	1.60	0.08	15.16	0.0074	0.049
10	0.80	0.08	9.93	0.0078	0.0294
5	0.40	0.08	7.17	0.0080	0.023
2	0.16	0.08	5.47	0.0083	0.0194
1	0.08	0.08	4.89	0.0084	0.0114
0.5	0.04	0.08	4.61	0.0084	0.0114
0.1	0.008	0.08	4.38	0.0084	0.0115

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